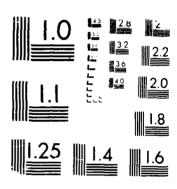
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## <u>Discrete Generalized Networks:</u> Application to Integer Constrained Flow Problems

Final Report

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Sanjiv Sarin

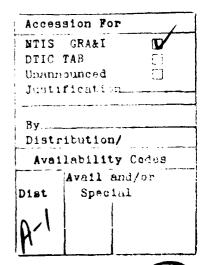
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Balasubramanian Ram

October 15, 1987

U.S. Army Research Office

Grant No. DAAG29-85-G-0086



North Carolina Agricultural and Technical State University



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The views, opinions, and/or findings contained in this report are those of the author and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

#### 1. Problem Statement

Consider a directed network G = (N,A) where N is the set of nodes and A is the set of arcs and |N| = n and |A|Let  $a_{ij}$  denote the arc multiplier for arc (ij)  $\epsilon$  A. effect of the arc multipliers is to transform the flow emanating from an originating node before it enters the consequent node, i.e., if a flow of  $f_{ij}$  leaves node i,  $a_{ij}f_{ij}$  is received at node j. Clearly, if  $a_{ij} > 1$ , flow is amplified and if  $a_{ij} < 1$ , flow is diminished while moving along arc (ij). We assume that  $a_{ij} > 0$  for all (i,j)  $\epsilon$  A since  $a_{i,j} = 0$  simply eliminates the arc from the network while  $a_{i,j} < 0$  causes a reversal in flow direction. Let  $l_{i,j}$ and  $c_{i,j}$  represent the lower and upper bounds to flow on  $\mbox{arc}$ (ij). A network as defined above with one or more  $a_{ij} \neq 1$ is referred to as a Generalized Network (GN). Note that when all  $a_{i,j} = 1$ , a GN reduces to the special case of a pure network. Mathematically, a GN can be represented as follows:

where s = source node
 t = sink node

S = set of immediate successor nodes to node i.
P; = set of immediate predecessor nodes to node i.

The generalized network structure is essentially a linear program whose coefficient matrix, ignoring simple bounds on variables, contains at most two non-zero entries in each column. Many problems, for which no pure network formulations are possible, can be modeled as generalized networks. Applications reported in literature include water distribution, copper refining process, cash management, blending, caterer problem, crew scheduling and manpower training.

When flow variables are required to be integers, the resulting problem is termed Discrete Generalized Network (DGN). The DGN structure has been used to model problems such as scheduling variable length television commercials into time slots, assigning jobs to computers in computer networks, designing communication networks and capital allocation problems (see Glover et al. [2] for these references). The primary objective of this project has been to identify solution methodologies for problems having the DGN structure. Some secondary outcomes of the research include a program for generating random networks and an application of DGN formulation to model an important problem in flexible manufacturing systems.

#### 2. Summary of Important Results

Results obtained from the research are summarized in the following six sections.

## 2.1 Solving Generalized Flow Network Problems with Positive Lower Bounds on Arc Capacities

A linear programming relaxation based branch and bound procedure was developed for solving the integer maximum flow generalized network problem. In solving the linear relaxation, an algorithm suggested by Pulat and Elmaghraby [5] was used. The complexity of their algorithm is a polynomial function of the number of nodes and number of arcs in the network. However, for it to be useful in an enumerative procedure, their algorithm had to be modified to handle non-zero lower bounds on arc flows.

This extension was the major contribution of a Masters Thesis by a graduate student, Mr. Barbaros Ozdemir. The modified algorithm preserves the computational attractiveness while making it more general. In this thesis, the linear relaxation technique was integrated within a tree search procedure and some random problems were solved. A copy of the thesis has been submitted to the U.S. Army Research Office. A summary of Mr. Ozdemir's thesis can be found in Ozdemir et al. [4].

## 2.2 Constraint Aggregation for Solving Discrete Generalized Network Problems.

Methods of constraint aggregation can be used for reducing the complexity of the discrete generalized network problem since all constraints (except for simple lower and

upper bounds) are equality constraints. A recently proposed method of constraint aggregation (Ram et al. [7] was used to perform the aggregation. The resulting problem possesses the knapsack structure with general bounds on variables.

A graduate student, Mr. Mohamed Qasim, worked on his Masters Thesis in solving this problem. His efforts resulted in (i) extension of the well know Balas and Zemel's [1] algorithm for solving the linear knapsack problems to the case of knapsacks with general bounds and (ii) development of a FORTRAN code for fully solving general bounded integer knapsack problems. A copy of Mr. Qasim's thesis has been submitted to the U.S. Army Research Office.

#### 2.3 Binary Expansion of Integer Variables

Binary expansion has found considerable use in integer programming since it is computationally simpler to deal with zero-one variables than with general bounded variables. A simple expression for binary expansion which uses the least number of binary variables was proposed. The advantage of the suggested formula is that it does not require any additional constraints. This result has been expressed in Ram and Sarin [8] and a copy of the manuscript has been submitted to the U.S. Army Research Office.

#### 2.4 A Program for Generating Connected Networks

In order to test the computational effectiveness of an

algorithm, it is necessary to have a program for generating random test problems. Currently known network generators such as NETGEN, NETGEN II and GENNI do not guarantee connectivity of the network. A new generator was proposed and a computer program was developed for creating network problems. The generated problems had one source node (one incoming arc), one sink node (one outgoing arc) and no transhipment nodes with demand or supply. These networks are of the maximum flow or minimum cost flow problems. An important feature of the generator is that it ensures connectivity of the entire network. The output of the program is in the form of the constraint matrix of the corresponding mathematical program.

The program has been fully described in Qasim et al. [6] and a copy of the manuscript has been submitted to the U.S. Army Research Office.

## 2.5 Application of Discrete Generalized Networks to a Problem in Flexible Manufacturing Systems.

An important problem is planning the operations of a Flexible Manufacturing System (FMS) is that of "machine loading and tool allocation". The specific problem is to equip the machines with proper tools in the tool magazine of limited capacity at the machine and to route the parts through the system so that the preselected system performance criteria are optimized. Typical criteria include minimum total machining time, minimum total

machining cost, minimum makespan, and minimum disparity in utilization of different machines.

Several variations of this problems have been identified and different solution approaches suggested for machine loading and tool allocation in FMS. The problem addressed is stated in Sarin and Chen [11]; the system performance criterion used by them is minimum total machining cost. A formulation is proposed for the problem within the framework of discrete generalized networks. In addition, an efficient solution procedure for the model is also presented. The model and solution procedure is applied to the example problem described in Sarin and Chen [11].

The machine loading and tool allocation problem was modeled as a discrete generalized network with "sum-to-one" side constraints. A branch and bound procedure was developed for determining a solution to the model and a FORTRAN code was written for implementing the ideas on a computer. Details of the modeling approach and the algorithm are provided in Ram et al. [9]. A copy of this manuscript has been submitted to the U.S. Army Research Office.

## 2.6 Objective Function Value Search for Equality Constrained Integer Programs.

As mentioned in Section 2.2, the discrete generalized network problem is an integer programming problem with equality constraints. In the last part of the project,

research was initiated on a new approach for solving equality constrained integer programs using constraint aggregation. A difficulty in using constraint aggregation is that round-off errors propagate rapidly when implementing the approach on a computer. This makes it difficult to identify feasible solutions since it involves checking for equality of two real numbers, both of which may be irrational. In this part of the project, instead of using constraint aggregation directly, a method is suggested that involves single-constraint subproblems that have irrational coefficients in the objective function. The constraint set includes the original objective function. Through an iterative search procedure, the optimal solution is found.

The famous algorithm of Martello and Toth [3] has been modified to handle non-integer coefficients in the objective function. Preliminary results with small problems indicate that the approach may work well with larger problems also. This study is being done as a Masters Thesis by Mr. Phillip Mintz, a graduate student. An interim report is available from the authors Ram et al. [10]. The final report and a copy of Mr. Mintz's thesis will be submitted to the U.S. Army Office when the student completes the study.

#### 3. Publications/Technical Reports

(i) Ozdemir, B., S.P. Pulat and S. Sarin, "Solving Generalized Flow Network Problems with Positive Lower Bounds on Arc Capacities", Working Paper, North Carolina A&T State University, Greensboro, NC 1987.

- (ii) Ram, B. and S. Sarin, "Binary Expansion of Integer Variables", Technical Report, North Carolina A&T State University, Greensboro, NC 1987, submitted for publication in the Journal of the Operational Research Society.
- (iii) Qasim, M., B. Ram and S. Sarin, "A Program for Generating Connected Networks", Technical Report, North Carolina A&T State University, Greensboro, NC 1987, submitted for publication in the International Journal of Mathematical and Management Sciences.
- (iv) Ram, B., S. Sarin and C.S. Chen, "A Model and A Solution Approach for the Machine Loading and Tool Allocation Problem in a Flexible Manufacturing System," Technical Report, North Carolina A&T State University, Greensboro, NC 1987, submitted for publication in the International Journal of Froduction Research.
- (v) Ram, B., S. Sarin and P. Mintz, "An Objective Function Value Search Method for Equality Constrained Integer Programs", Technical Report, North Carolina A&T State University, Greensboro, NC 1987.

#### 4. Participating Scientific Personnel

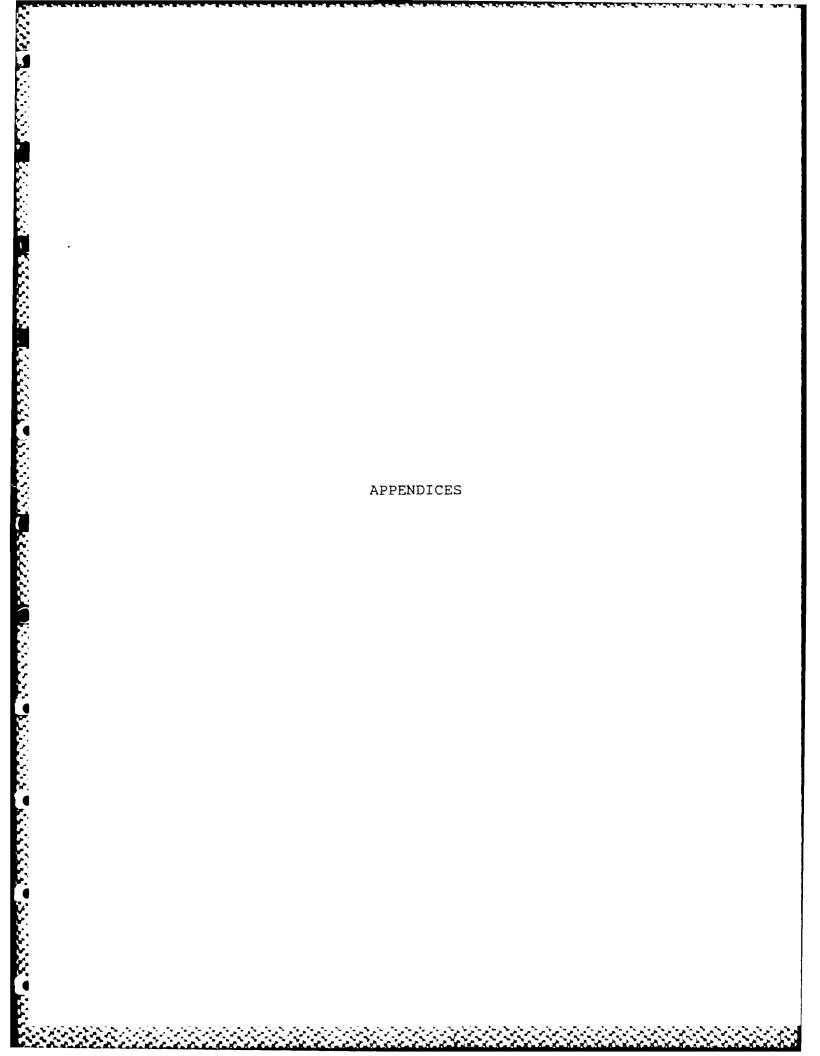
- Dr. P.S. Pulat Assistant Professor (4/15/85 - 7/1/85)
- 2. Dr. B. Ram Associate Professor (5/8/86 10/14/87)
- Dr. S. Sarin Assistant Professor (4/15/85 - 10/14/87)
- 4. Mr. B. Ozdemir Graduate Student (4/16/85 12/13/85)
- 5. Mr. M. Qasim Graduate Student (1/1/86 - 7/19/86)
- 6. Mr. P. Mintz Graduate Student (1/15/87 7/15/87)

#### Degrees Awarded

- (1) Mr. Barbaros Ozdemir Master of Science in Industrial Engineering. May 1987. (Copy of Thesis Title and Abstract included in Appendices).
- (2) Mr. Mohamed Qasim Master of Science in Industrial Engineering. May 1987. (Copy of Thesis Title and Abstract included in Appendices).

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- [1] Balas, E. and E. Zemel, "An Algorithm for Large Zero-one Knapsack Problems", Operations Research, 28(5), 1130-1154, 1980.
- [2] Glover, F., J. Hultz, D. Klingman and J. Stutz, "Generalized Networks: A Fundamental Computer-Based Planning Tool", <u>Management Science</u>, 24(12), 1209-1220, 1978.
- [3] Martello, S. and P. Toth, "Algorithm for Solving the 0-1 Knapsack Problem", Computing, 21, 81-86, 1978.
- [4] Ozdemir, B., S.P. Pulat and S. Sarin, "Solving Generalized Flow Network Problems with Positive Lower Bounds on Arc Capacities", Working Paper, North Carolina A&T State University, Greensboro, NC, 1987.
- [5] Pulat, P.S. and S.E. Elmaghraby, "On Maximizing Flow in Generalized Flow Networks", OR Report No. 202, North Carolina State University, Raleigh, NC, 1984.
- [6] Qasim, M., B. Ram and S. Sarin, "A Program for Generating Connected Networks", Technical Report, North Carolina A&T State University, Greensboro, NC, 1987.
- [7] Ram, B., M.H. Karwan and A.J.G. Babu, "Aggregation of Constraints In Integer Programming", <u>European Journal of Operational Research</u>, 1987 (To appear).
- [8] Ram, B. and S. Sarin, "Binary Expansion of Integer Variables", Technical Report, North Carolina A&T State University, Greensboro, NC 1987.
- [9] Ram, B., S. Sarin and C.S. Chen, "A Model and a Solution Approach for the Machine Loading and Tool Allocation Problem in a Flexible Manufacturing System", Technical Report, North Carolina A&T State University, Greensboro, NC, 1987.
- [10] Ram, B., S. Sarin and P. Mintz, "An Objective Function Value Search Method for Equality Constrained Integer Programs", Technical Report, North Carolina A&T State University, Greensboro, NC, 1987.
- [11] Sarin, S.C. and C.S. Chen, "Machine Loading and Tool Allocation Problem in a Flexible Manufacturing System", International Journal of Production Research, 25,7,1081-1094, 1987.



#### MAXIMUM FLOW IN GENERALIZED INTEGER NETWORKS:

ЬУ

#### Barbaros Turgut OZDEMIR

A thesis submitted to the Graduate Faculty of North Carolina Agricultural and Technical State University in partial fulfillment of the requirements for the Degree of Master of Science in Industrial Engineering.

Greensboro, North Carolina 1986

Approved by:

Advisor

Chairperson of the Department

Dean of the School Of Graduate Studies

#### **ABSTRACT**

In this research, we propose a procedure for solving Generalized Integer Flow Networks. Applications of the Generalized Integer Flow Networks include machine loading problems, water reservoir systems, investment decisions. A polynomically bounded algorithm to solve linear Generalized Flow Networks was recently proposed by Pulat [1984]. In this thesis an extension to this algorithm is developed to handle non-zero lower bounds. The extension permits the incorporation of this algorithm as a bounding technique in using a branch-and-bound algorithm for solving Generalized Integer Networks.

#### ACKNOWLEDGEMENTS

I would like to empress my grateful appreciation to my theory advisors. Dr. Pakize Simin Pulat and Dr. Sanjiv Sarim, for their support and assistance during the preparation of this thesis. I would also like to thank my friends Raif Onvural and Avdin Yardim for their continuos support and precious time.

Grateful acknowledgement goes to the U.S Army research office.

Research Triangle Park for making this research possible, through Grant # DANGGY-85-G-0086.

I also would like to thank my parants for their endless help and support.

## SOLVING INTEGER GENERALIZED NETWORK PROBLEMS BY CONSTRAINT AGGREGATION

Ву

Mohamed M. Qasim

A Thesis Submitted to the Graduate Faculty of North Carolina Agricultural and Technical State University in partial fulfillment of the requirements for the Degree of Master of Science in Industrial Engineering.

Greensboro, North Carolina

1987

Approved by:

Advisor

Chairperson of the Department

Dean of the School of Graduate Studies

#### Abstract

In this study we propose a computational procedure for solving generalized network flow problems with integral flows. The method presented is different from earlier approaches in that, the mathematical programming formulation is solved directly as an integer program instead of resorting to network techniques. The problem size is reduced using constraint aggregation. The aggregated problem has the form of the equality knapsack problem with bounded integer variables. The algorithm of Balas and Zemel is modified to deal with bounded variables.

The algorithm is coded in FORTRAN and is tested on randomly generated problems. Computational results are also presented in this report.

#### ACKNOWLEDGMENTS

I would like to express my deepest thanks to Allah for providing me with all resources necessary to complete my work. My grateful appreciation goes to my thesis adviser, Dr. Sanjiv Sarin, for his helpful guidance, encouragement, and suggestions during the preparation of this thesis. The guidance and suggestions given by Dr. Balasubramanian Ram were very helpful as well.

Grateful acknowledgment is made to the United States Army Research Office, Grant No. DAAG29-85-G-0086 for supporting this research.

And last, I cannot forget to thank my family, who supported my education and made it possible for me to write this thesis. To them I owe my appreciation and love.

## Solving Generalized Flow Network Problems with Positive Lower Bounds on Arc Capacities

by:

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Simin P. Pulat\*\*
and
Sanjiv Sarin\*

\*North Carolina Agricultural and Technical State University

\*\* University of Oklahoma

February 1987

This research was supported in part by the U.S. Army Research Office Grant NO: DAAG29-85-G-0086. This support is gratefully acknowledged.

Department of Industrial Engineering
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Greensboro, NC 27411

#### Abstract

The Generalized Flow Network (GFN) problem involves maximizing flow through a network which allows gain or loss of flow across its arcs. Several important applications of GFN have been cited in the literature. This paper focuses attention on a recent method for solving GFN's. We present an extension to the algorithm which allows solving GFN problems with non-zero lower bounds on arc capacities.

#### F12W-1/15

Network Optimization, Network Flow, Generalized Networks.

### Binary Expansion of Integer Variables

bу

Balasubramanian Ram

and

Sanjiv Sarin

#### Acknowledgment :

The research reported here was supported in part by the US Army Research Office, Grant Number DAAG29 - 85 - G - 0086. This support is gratefully acknowledged.

April 28, 1987

Department of Industrial Engineering North Carolina A&T State University Greensboro, NC 27411, USA

#### Abstract

A bounded integer variable can be substituted by a sum of binary variables. We present a new expression for performing this expansion.

<u>Keywords</u>: Integer Programming, Binary Expansion

#### A Program for Generating Connected Networks

by

M. QasimB. RamS. Sarin

#### Acknowledgment:

The research reported here was supported in part by the US Army Research Office, Grant Number DAAG29 - 85 - G - 0086. This support is gratefully acknowledged.

August 1987

Department of Industrial Engineering North Carolina A & T State University Greensboro, North Carolina

#### Abstract

We present a program for generating random network problems. The generated problems are of the maximum flow or minimum cost type. This generator may be used for computationally studying the effectiveness of network optimization algorithms.

#### Keywords

Network Optimization, Test Problem Generator

# A Model and A Solution Approach For The Machine Loading and Tool Allocation Problem In A Flexible Manufacturing System

by

Balasubramanian Ram Sanjiv Sarin C. S. Chen

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August 1987

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#### Abstract

We consider the problem of planning machine loading and tool allocation in a flexible manufacturing system. The problem is modeled as a discrete generalized network with simple side constraints. An algorithm is described to yield a solution to this problem. An important aspect of the modeling process presented here is its ease of application to other planning problems in flexible manufacturing systems.

## AN OBJECTIVE FUNCTION VALUE SEARCH METHOD FOR EQUALITY-CONSTRAINED INTEGER PROGRAMS

by

Balasubramanian Ram Sanjiv Sarin Phillip Mintz

<u>Acknowledgement</u>: The research reported here was supported in part by the US Army Research Office, Grant Number DAAG29-85-G-0086. This support is gratefully acknowledged.

August 1987

Department of Industrial Engineering North Carolina A&T State University Greensboro, NC 27411

#### Abstract

We consider an approach for solving equality-constrained integer programs. It involves a search over an easily-computed range of objective function values. The search involves solving single constraint integer programs.

-/LME